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I ♥ My Attitude Problem

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GODDARD SPACE FLIGHT CENTER

Outline

- Algorithms
  - Kalman Filters
  - Wahba's Problem
- Spacecraft
  - SMM
  - SAMPEX
  - HST
  - TRMM
  - WMAP

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History

- AIAA 76-1910 "Dynamic Modeling for Attitude Determination," E. J. Lefferts and F. L. Markley

History

- AIAA 78-1245 "Application of Square-Root Filtering for Spacecraft Attitude Control," J. A. Sorenson, S. F. Schmidt, and T. Goka
- AIAA 78-1246 "Real Time Precision Attitude Determination System (RETPAD) for Highly Maneuverable Spacecrafts," K. Yong and R. P. Headley
- AIAA 78-1247 "Attitude Control Algorithms for the Solar Maximum Mission," F. L. Markley
- AIAA 78-1248 "Precision Attitude Determination for Multimission Spacecraft," J. W. Murrell
- AIAA 78-1249 "Approximate Algorithms for Fast Optimal Attitude Computation," M. D. Shuster
- AIAA 78-1250 "Interferometric Attitude Determination with the Global Pos. Sys." J. F. Ellis and G. A. Creswell

Steady-State Kalman Filter

- 1D attitude dynamics with gyro in model replacement mode

$$\frac{d}{dt}\begin{bmatrix}\theta\\ b\end{bmatrix}=\begin{bmatrix}\omega_g\\ 0\end{bmatrix}+\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ b\end{bmatrix}+\begin{bmatrix}-n_v\\ n_u\end{bmatrix}$$

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- Angle measurements at time interval Δt with NEA σ_n
  - Assume Δt << filter time constant

$$\frac{dP}{dt}=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}P+P\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix}^T+\begin{bmatrix}\sigma_v^2 & 0\\ 0 & \sigma_u^2\end{bmatrix}-P\begin{bmatrix}(\sigma_n^2\Delta t)^{-1} & 0\\ 0 & 0\end{bmatrix}P=0$$

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$$P_{\theta\theta} = \sigma_n[\sigma_v^2\Delta t + \sigma_n\sigma_u(\Delta t)^{3/2}]^{1/2}, \qquad P_{\theta b} = -\sigma_n\sigma_u(\Delta t)^{1/2},$$

$$P_{bb} = \sigma_u[\sigma_v^2 + 2\sigma_n\sigma_u(\Delta t)^{1/2}]^{1/2}$$

Farrenkopf's Analysis

- "Analytic Steady-State Accuracy Solutions for Two Common Spacecraft Attitude Estimators," *JGC*, July-August 1978

$$P_{\theta\theta}(\mp) = \zeta^{\pm 1}(\zeta - \zeta^{-1})\sigma_n^2, \qquad P_{\theta b}(\mp) = -\zeta^{\pm 1}\sigma_u\sigma_n(\Delta t)^{1/2},$$

$$P_{bb}(\mp) = \sigma_u[\sigma_v^2 + 2\bar{\sigma}\sigma_u(\Delta t)^{1/2} + \tfrac{1}{3}\sigma_u^2(\Delta t)^2]^{1/2} \pm \tfrac{1}{2}\sigma_u^2\Delta t$$

$$\zeta = \sigma_n^{-1}\left\{\bar{\sigma} + \tfrac{1}{4}\sigma_u(\Delta t)^{3/2} + \tfrac{1}{2}[\sigma_v^2\Delta t + 2\bar{\sigma}\sigma_u(\Delta t)^{3/2} + \tfrac{1}{3}\sigma_u^2(\Delta t)^3]^{1/2}\right\}$$

$$\bar{\sigma} \equiv [\sigma_n^2 + \sigma_e^2 + \tfrac{1}{4}\sigma_v^2\Delta t + \tfrac{1}{48}\sigma_u^2(\Delta t)^3]^{1/2}$$

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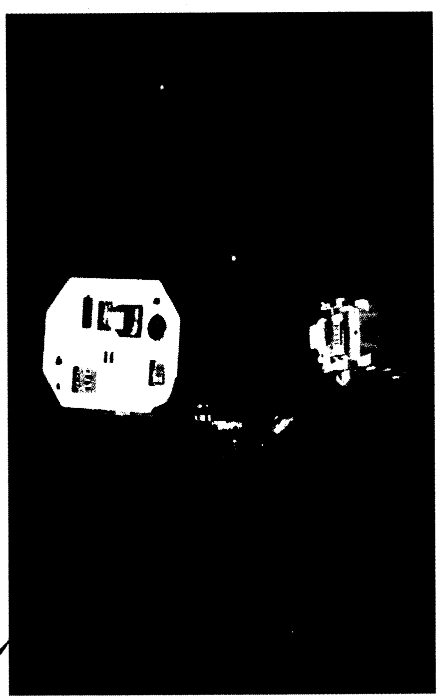
- "Kalman Filtering for Spacecraft Attitude Estimation," 1982

SMM



- Mass = 2315 kg
- Pointing = 5 arcsec
- Launch (Feb 1980)
- Spin stabilized (Jan 1981)
  - Henry Hoffman
  - Jim Donohue
  - Tom Flatley
  - "Satellite Doctors"
- Repaired (April 1984)
- Reentered (Dec 1989)

Wahba's Problem (1965)

$$L \equiv \tfrac{1}{2}\sum_i a_i \left|\mathbf{b}_i - A\mathbf{r}_i\right|^2 = \lambda_0 - \mathrm{trace}(AB^T),$$

$$\text{where } \lambda_0 \equiv \sum_i a_i \text{ and } B \equiv \sum_i a_i \mathbf{b}_i \mathbf{r}_i^T$$

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Davenport's q method:

$$= \lambda_0 - q^T K q, \text{ where } K \equiv \begin{bmatrix} B + B^T - I_{3\times 3}\,\mathrm{trace}(B) & \sum_i a_i \mathbf{b}_i \times \mathbf{r}_i \\ \left(\sum_i a_i \mathbf{b}_i \times \mathbf{r}_i\right)^T & \mathrm{trace}(B) \end{bmatrix}$$

so $\quad Kq_{\mathrm{opt}} = \lambda_{\max} q_{\mathrm{opt}}$

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so $\quad Kq_{\mathrm{opt}} = \lambda_{\max} q_{\mathrm{opt}}$

$$L = \lambda_0 - \lambda_{\max} \quad \Rightarrow \quad \lambda_{\max} \approx \lambda_0 \quad \Rightarrow \quad \text{QUEST}$$

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FOAM:

$$A_{\mathrm{opt}} = (\kappa\lambda_{\max} - \det B)^{-1}[(\kappa + \|B\|_F^2)B + \lambda_{\max}\mathrm{adj}B^T - BB^T B],$$

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"25 Years of Wahba's Problem"

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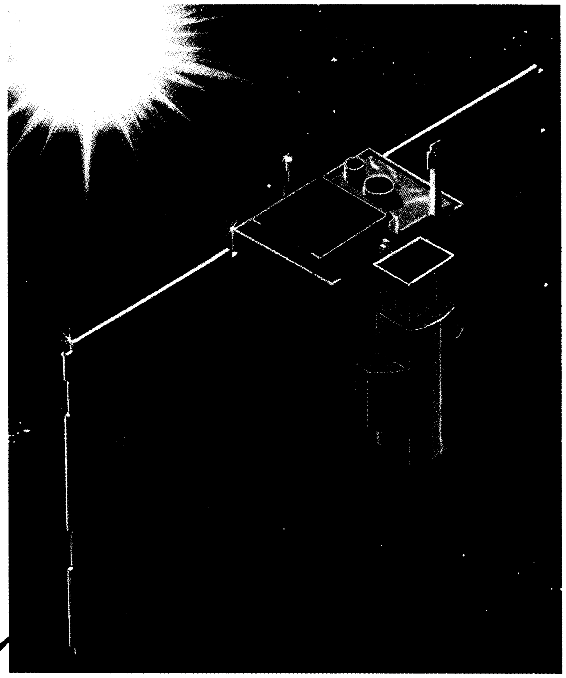
"30 Years of Wahba's Problem"

SAMPEX (first SMEX mission)



- Proposal (Sept 1988)
- Launch (July 1992)
- Mass = 160 kg
- Pointing = 2°
- Sensors
  - Sun sensor (0.5°)
  - Magnetometers
- Actuators
  - One reaction wheel
  - Magnetic torquers
- 80386 processor

SAMPEX ADACS

- Largely developed by Tom Flatley
- Attitude matrix A from TRIAD
  - Onboard ephemeris model
  - Onboard magnetic field model (IGRF)
- Magnetic control law $\mathbf{m} = k_{\text{mag}}(\Delta\mathbf{H} \times \mathbf{B})$

$$\Delta\mathbf{H} = (\mathbf{H} - H_0\mathbf{j}) + (\mathbf{H} - H_0\mathbf{s}), \quad \mathbf{H} = I\boldsymbol{\omega} + H_{\text{wheel}}\mathbf{j}, \quad [\boldsymbol{\omega}\times] = -\dot{A}A^T$$

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- Needed a simple Kalman filter

$$\mathbf{H} = (1-K)\mathbf{H}_{\text{predicted}} + K\mathbf{H}_{\text{derived}}$$

$$\mathbf{H}_{\text{predicted}}(t) = A(t)A^T(t-\Delta t)\mathbf{H}(t-\Delta t) + (\mathbf{m}\times\mathbf{B})\Delta t$$

SAMPEX ADACS

The Solar, Anomalous and Magnetospheric Particle Explorer (SAMPEX)

has located a radiation belt that lies inside two Van Allen belts. Russian and U.S. scientists concluded in 1991 that there was another belt, but could not locate it. SAMPEX was launched in July 1992 on a Scout launcher from Vandenberg Air Force Base, Calif.

Small Explorer Satellite Finds Radiation Belt

By VINCENT KIERNAN
Special to Space News

BALTIMORE — A NASA satellite has pinpointed a belt of particles of interstellar matter circling the Earth, a development that may vindicate the space agency's recent moves toward using smaller and less expensive spacecraft to conduct space science.

The Solar, Anomalous and Magnetospheric Particle Explorer (SAMPEX) found that the radiation belt lies inside the two Van Allen radiation belts, which have been known to surround the planet since 1958.

The belt that SAMPEX studied is composed of electrically charged particles known as anomalous cosmic rays. The particles are atoms of interstellar gas that have been given an electrical charge during their travels through the solar system, said SAMPEX researcher Richard Mewaldt of the California Insti-

likely is debris from exploding supernovas, while other material may date to the Big Bang cataclysm that is believed to have produced the universe, he said.

"It's sort of a collecting pot, a melting pot, for everything that has happened in the galaxy for the last 10 or 15 billion years," Mewaldt said.

So far, SAMPEX has detected atoms of nitrogen, oxygen and neon, he said. Eventually, researchers expect SAMPEX to identify other substances, such as carbon, krypton and argon, he said.

SAMPEX's orbit varies from 342 miles to 419 miles above the Earth. At those altitudes, the belt of interstellar matter is most intense above the south Atlantic Ocean, between the southern reaches of Africa and South America.

Russian and American scientists had deduced the existence

Spectrometer Telescope, which was built by the California Institute of Technology.

Lennard Fisk, NASA's chief scientist, told reporters the SAMPEX data are the first fruits from NASA's Small Explorer program, begun in 1988 to field inexpensive scientific satellites on lower-cost, small boosters.

NASA has firm plans for two further Small Explorer spacecraft: the Fast Auroral Snapshot Explorer, slated for launch in September 1994, and the Submillimeter Wave Astronomy Satellite, scheduled for a 1995 launch.

Fisk said NASA officials are weighing proposals for further satellites in the series.

Meanwhile, researchers at the geophysicists' conference said they expect SAMPEX also to provide valuable insights into behavior of the protective ozone that shields the Earth's surface from ultraviolet radiation.

Scientists believe the ozone is being attacked by chemicals released into the atmosphere by humans, but ozone levels may fluctuate naturally because of high-energy electrons striking the upper atmosphere, said Linwood Callis, an atmospheric scientist at NASA's Langley Research Center, Hampton, Va.

SAMPEX will measure the levels of incoming electrons in an effort to estimate how much of the ozone depletion is due to pollution and how much is due to the electrons, he said.

Japan To Propose Global

HST

- Mass = 11,000 kg
- Pointing = 7 mas
- STS-31 release (April 1990)

HST

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- Pointing = 7 mas
- STS-31 release (April 1990)

- SAGA
  - Henry Hoffman
  - Jim Donohue
  - MSFC
  - Lockheed

Zero Gyro Sunpoint (ZGSP)

- ZGSP
  - John Nelson and his Lockheed team
  - Points z axis using sun sensor angles and derived rates
  - Momentum bias on this axis holds attitude during eclipse
  - Only rate damping around the sun line

$$\omega_z = (B_y \dot{B}_x - B_x \dot{B}_y) \big/ (B_x^2 + B_y^2)$$

  - 38 consecutive days in Nov-Dec 1999

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  - 38 consecutive days in Nov-Dec 1999
- System momentum test to detect "soft" gyro failure

$$\mathbf{H} = I\boldsymbol{\omega} + \mathbf{H}_{\text{wheels}}$$

$$\mathbf{T}_{\text{dist}} = \dot{\mathbf{H}} + \boldsymbol{\omega} \times \mathbf{H} - \mathbf{m} \times \mathbf{B} - \mathbf{T}_{\text{gravity gradient}}$$

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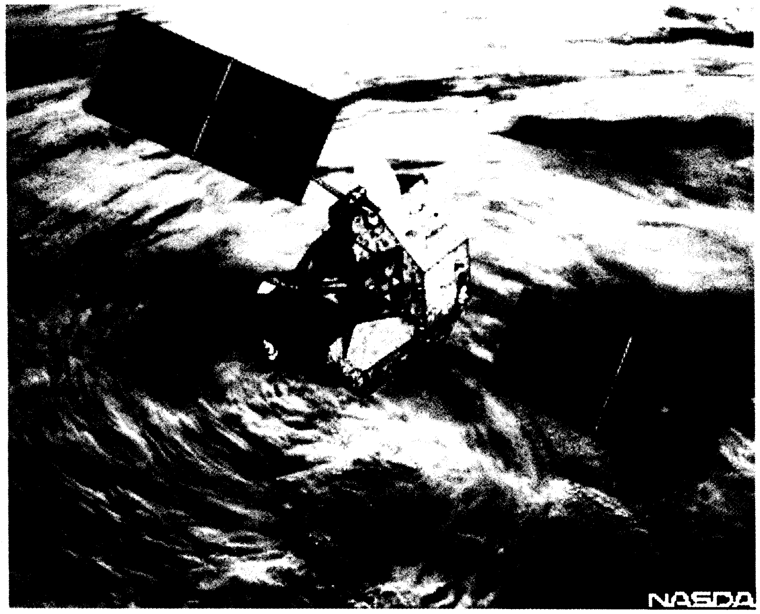
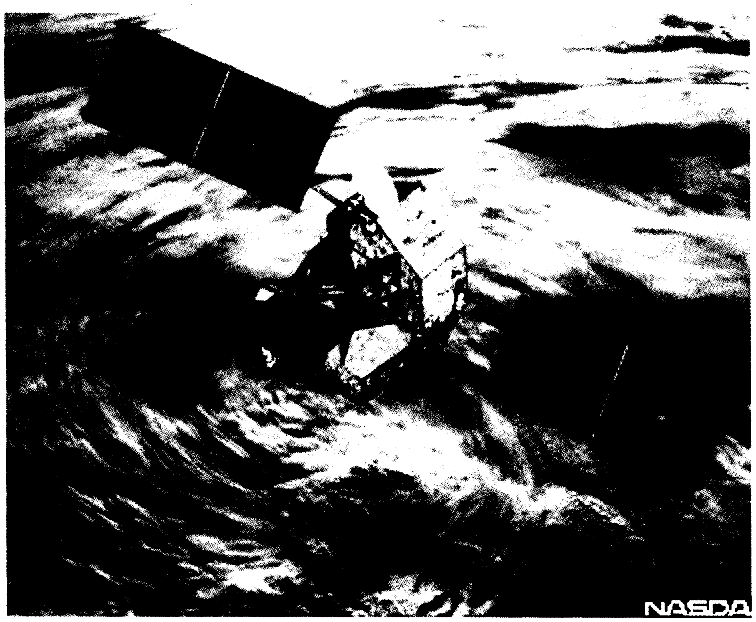
TRMM



- Mass = 3500 kg
- Pointing = 0.2°

TRMM



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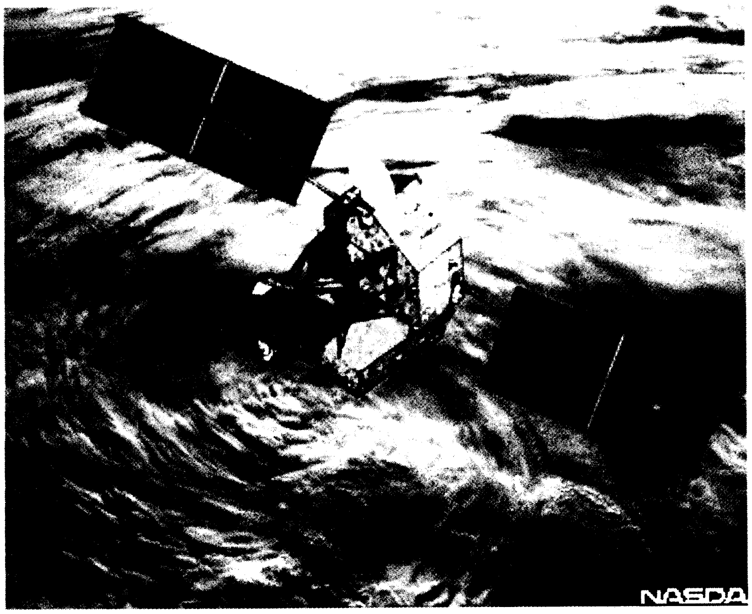
TRMM



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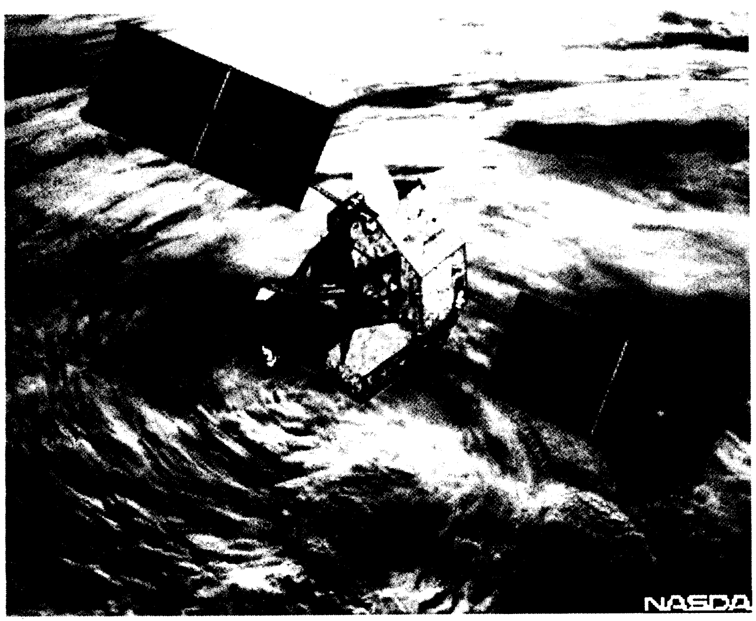
  - DMSP fogging

TRMM



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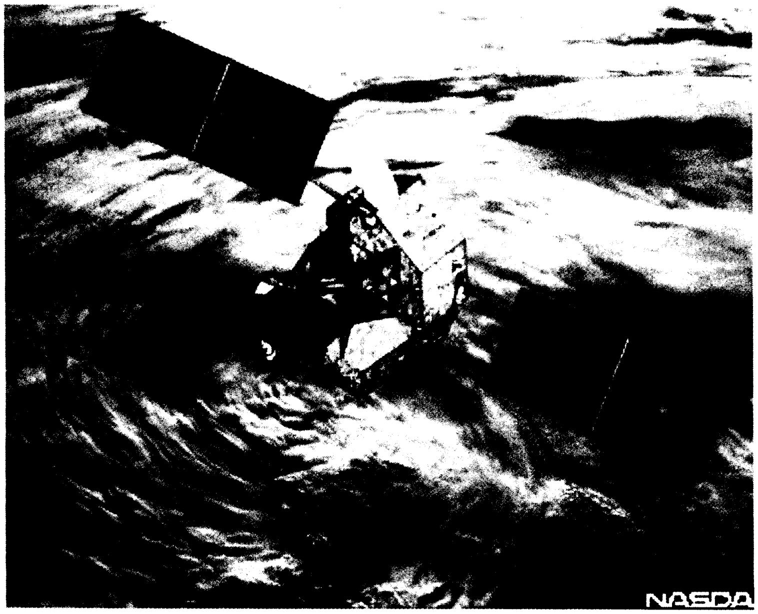
TRMM



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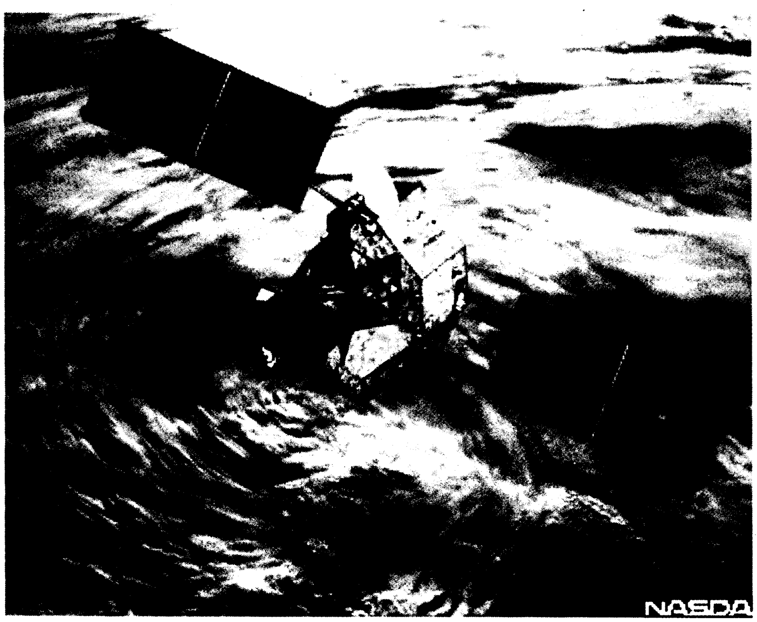
- Launch (1997)

TRMM



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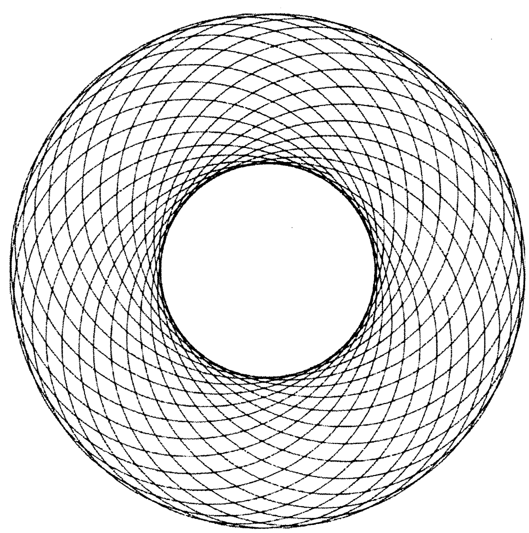
- Launch (1997)

- Orbit raising (2001)

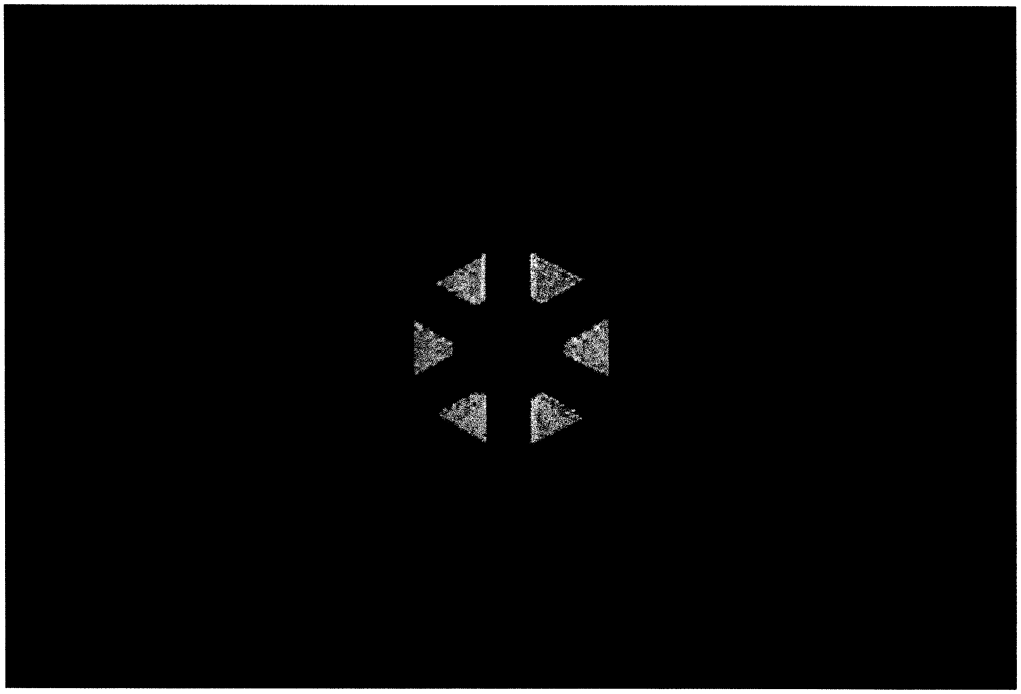
WMAP

- Proposal effort (Nov 1994)
  - Chuck Bennett
  - David Wilkinson
  - Cliff Jackson
- Earth-Sun L2
  - Passive cooling to 90 K
  - Infrequent thrusting
- Scan pattern
  - Spin axis 112.5° from Sun
  - Fast spin (0.5 rpm)
  - Slow precession (1 rph)

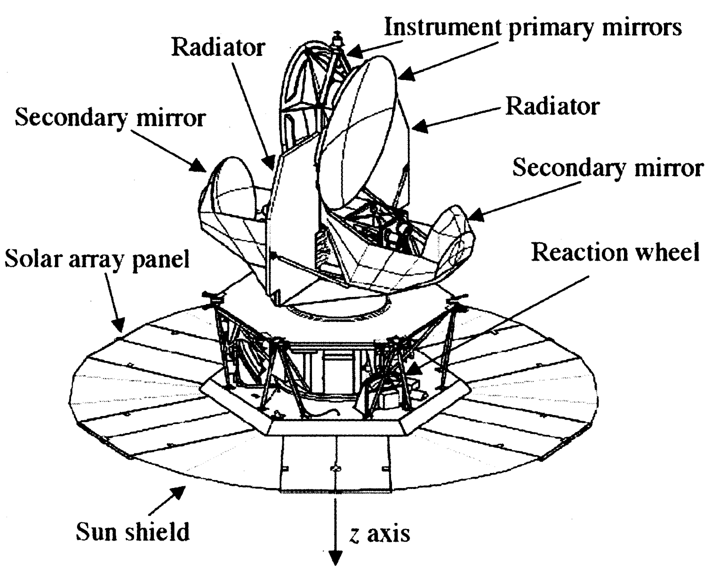
WMAP



WMAP Scan Pattern

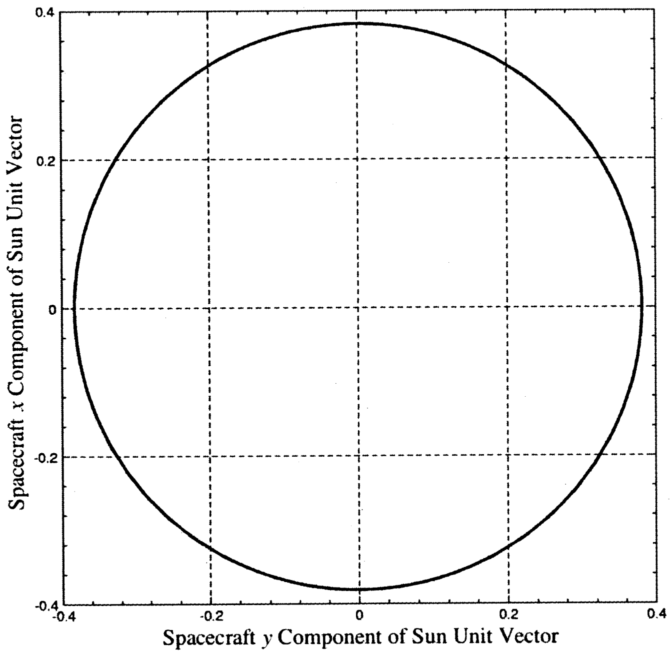


WMAP ADACS



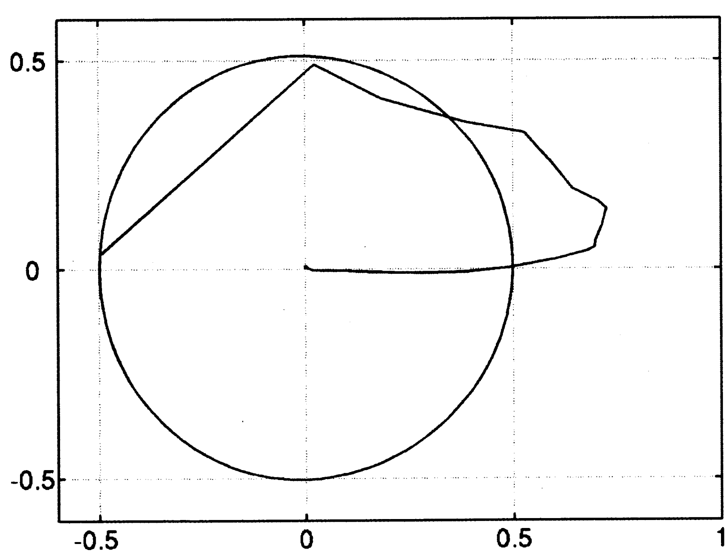
- Concept (Dec 1994)
  - Zero momentum
  - Reaction wheels
  - Star tracker
  - Sun sensors
  - TARAs
  - Kalman filter
- Mass = 830 kg
- Pointing = 0.03
- Launch (2001)

WMAP Attitude Anomaly



- The least interesting figure in any paper

WMAP Attitude Anomaly



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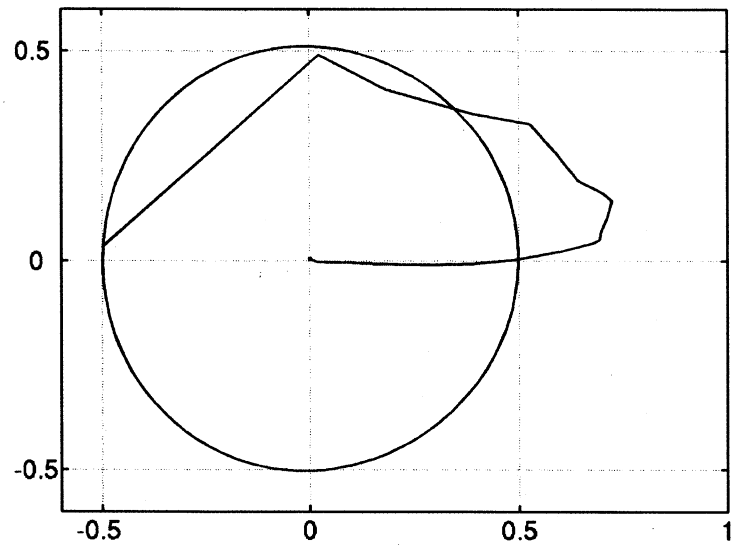
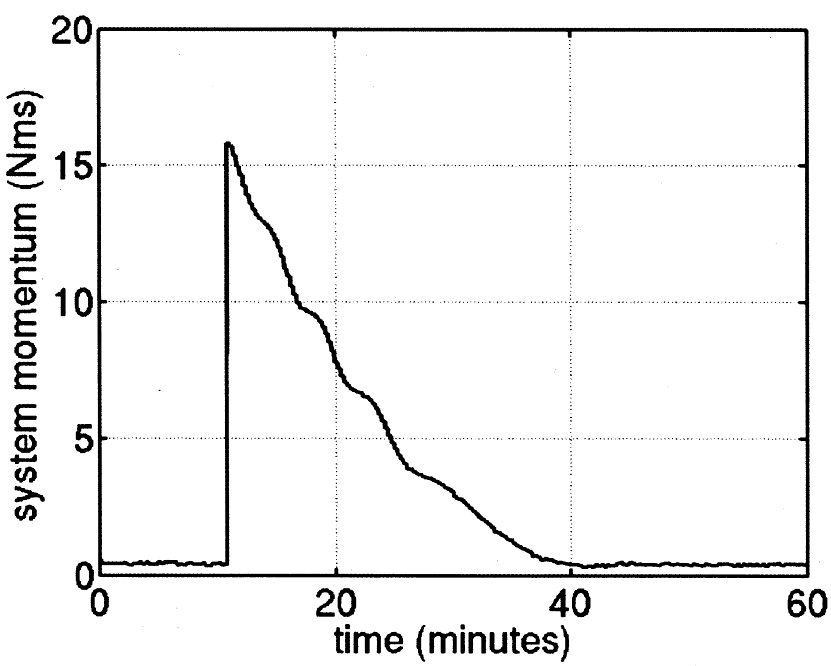
WMAP Attitude Anomaly



- The least interesting figure in any paper
- It became more interesting on February 17, 2005
- What happened?

WMAP Attitude Anomaly



Summary

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- Even though all the interesting problems were solved in ~~1969~~ ~~1972~~ ~~1975~~ ~~1978~~ ~~1981~~ ~~1984~~ ~~1987~~ ~~1990~~ ~~1993~~ 1996

- We can still find interesting problems to solve